Tip. When simplifying radical expressions, it is helpful to rewrite a number using its prime factorization and cancel powers.

Example. $108 = 2^2 3^3$ so $\sqrt[3]{108} = \sqrt[3]{2^2 3^3} = 3\sqrt[3]{2^2} = 3\sqrt[3]{4}$

1. Evaluate each expression.

(a) $(-3)^4$ (b) -3^4 (c) $\left(\frac{1}{8}\right)^2 \cdot (-2)^3$

(d) $(-2)^5$ (e) -2^5 (f) $(-6)^2 \cdot \left(\frac{1}{6}\right)^2$

(g) $\left(\frac{2}{5}\right)^0 \cdot 3^{-1}$ (h) $\frac{3^{-2}}{5^0}$ (i) $\left(\frac{-2}{5}\right)^{-2}$

(j) $5^2 \cdot 5$ (k) $5^8 \cdot 5^{-6}$ (l) $5^{-8} \cdot 5^6$

(m) $\left(2^3\right)^2$ (n) $\frac{10^8}{10^5}$ (o) $\frac{10^2}{10^{-2}}$

2. Simplify.

(a) $\sqrt{12}$ (b) $\sqrt{18}$ (c) $\sqrt[3]{250}$

(d) $\sqrt[5]{243}$ (e) $\sqrt[5]{486}$ (f) $\sqrt[4]{162}$

(g) $\sqrt{\frac{27}{16}}$ (h) $\frac{\sqrt{18}}{\sqrt{36}}$ (i) $5\sqrt[3]{81}$

(j) $\sqrt{2} \cdot \sqrt{6}$ (k) $\sqrt{14} \cdot \sqrt{32}$ (l) $\frac{\sqrt{80}}{\sqrt{5}}$

(m) $\sqrt[3]{500}$ (n) $\sqrt[4]{24} \cdot \sqrt[4]{14}$ (o) $\frac{\sqrt{63}}{\sqrt{7}}$

(p) $\sqrt[3]{75} \cdot \sqrt[3]{20}$ (q) $\sqrt[3]{\frac{1}{3}} \cdot \sqrt[3]{\frac{1}{81}}$ (r) $\sqrt[5]{\frac{1}{2}} \cdot \sqrt[5]{\frac{1}{16}}$

(s) $\sqrt[6]{\frac{1}{4}} \cdot \sqrt[6]{256}$ (t) $\sqrt[3]{5} \cdot \sqrt[3]{135}$

3. Simplify each expression and eliminate negative exponents.

(a) $x^5 \cdot x^8$ (b) $(2x^3)^2$ (c) $x^{-3} \cdot x^5$

(d) $y^6 \cdot y^9$ (e) $(3x)^3$ (f) $y^7 \cdot y^{-3}$

(g) $z^{-8} \cdot z^3$ (h) $x^{-2}x^{-6}x^4$ (i) $\frac{x^{15}}{x^{10}}$

(j) $y^3 \cdot y^{-9}$

(k) $w^5 w^{-8} w^4$

(l) $\dfrac{x^8 x^0}{x^{12}}$

(m) $\dfrac{b^8 b^{-3}}{b}$

(n) $(z^3 z^5)^2$

(o) $(3x^4)\left(\dfrac{x}{3}\right)^3$

(p) $\dfrac{y^3 y^5}{y^2 y^{-3}}$

(q) $(-2b^3 b^3)^3$

(r) $(-3x^2)^2(2x^{-2})^3$

4. Simplify each expression and eliminate negative exponents.

(a) $\dfrac{xy}{7x^{-4}y^{-2}}$

(b) $\dfrac{7y^6}{4y^5 z^4}$

(c) $(x^3 y^{-5})(2x^{-4}y^2)(4xy^5)$

(d) $(xw)(6x^{-6}w^{-4})$

(e) $(w \cdot 4w^2 \cdot w^2)^3$

(f) $\left(\dfrac{y^2}{y}\right)^3$

(g) $(3x \cdot 4x^2)^3$

(h) $\left(\dfrac{2y^4}{4y}\right)^2$

(i) $\left(\dfrac{9z}{8z^6}\right)^3$

(j) $\dfrac{x^{-3}y^{-2}}{y^{-1}}$

(k) $\left(\dfrac{a^3 b^{-2}}{a^{-3}b^2}\right)^3$

(l) $\left(\dfrac{x}{y^2}\right)^5 \left(\dfrac{x^2 y^3}{z^2}\right)^3$

(m) $\dfrac{(a^{-1}b^3)^2}{(a^2 b^{-3})^3}$

(n) $\left(\dfrac{x^2 z^4}{2y^5}\right)\left(\dfrac{3x^2 y^3}{z^2}\right)^2$

(o) $\dfrac{(w^2 v)^3}{(w^2 v^{-3})^2}$

(p) $\dfrac{16x^3 y^{-5}}{4x^{-6}y^8}$

(q) $\left(\dfrac{w}{3x^{-3}}\right)^{-2}$

(r) $\left(\dfrac{2x^{-1}y}{x^{-3}y^{-2}}\right)^{-3}$

(s) $\left(\dfrac{a^{-2}b^{-1}c^{-2}}{b^{-5}c^2 a^{-4}}\right)^{-1}$

(t) $\left(\dfrac{x^4 y^{-2}}{5x^{-1}y}\right)^{-2}$

(u) $\left(\dfrac{2rs^{-2}t^{-3}}{3s^3 t^{-4} r^2}\right)^{-3}$

5. Express the following in the form x^r.

(a) $(\sqrt[5]{x})^6$ (b) $\sqrt[8]{x^3}$ (c) $\dfrac{1}{(\sqrt{x})^5}$ (d) $\dfrac{1}{\sqrt[3]{x^4}}$ (e) $\sqrt[4]{\sqrt[3]{x}}$ (f) $\sqrt{\dfrac{1}{\sqrt[5]{x}}}$

6. Express the following in the form x^r.

(a) $x^{\frac{5}{2}}x^3$ (b) $\dfrac{x^{\frac{6}{7}}}{x^4}$ (c) $(x^3)^{-\frac{4}{5}}$ (d) $x^{\frac{7}{5}}x^{-\frac{8}{3}}$ (e) $(x^{\frac{2}{3}})^{\frac{4}{9}}$ (f) $\dfrac{1}{x^{\frac{5}{2}}}$

(g) $\left(\dfrac{1}{x^3}\right)^{-\frac{2}{3}}$ (h) $\dfrac{1}{x\sqrt{x}}$ (i) $x^2(\sqrt[3]{x})$ (j) $\dfrac{x}{x^{\frac{2}{5}}}$ (k) $\dfrac{x^{\frac{1}{3}}}{x}$ (l) $\dfrac{1}{x^{-\frac{5}{4}}}$

7. Which of the following equations are true for all nonzero values of x and y?

(a) $x^5 + x^2 = x^7$ (b) $x^4x^7 = x^{11}$ (c) $(xy)^3 = x^3y^3$ (d) $(x^4)^3 = x^7$

(e) $(x+y)^4 = x^4 + y^4$ (f) $(x^3)^3 = x^9$ (g) $x^7 - x^3 = x^4$ (h) $\dfrac{x^8}{x^2} = x^4$

(i) $\dfrac{x^4}{y^4} = \left(\dfrac{x}{y}\right)^4$ (j) $\dfrac{x^9}{x^3} = x^6$ (k) $(x-y)^4 = \dfrac{x^4}{y^4}$ (l) $\dfrac{x^{10}}{y^2} = \left(\dfrac{x}{y}\right)^5$

8. Evaluate each expression.

(a) $32^{\frac{1}{5}}$ (b) $(-8)^{\frac{1}{3}}$ (c) $(-1)^{\frac{1}{3}}$ (d) $81^{\frac{1}{4}}$

(e) $\left(\dfrac{1}{81}\right)^{\frac{1}{4}}$ (f) $32^{\frac{2}{5}}$ (g) $\left(\dfrac{8}{27}\right)^{-\frac{1}{3}}$ (h) $\left(\dfrac{125}{64}\right)^{\frac{2}{3}}$

(i) $\left(\dfrac{81}{25}\right)^{-\frac{1}{2}}$ (j) $\left(\dfrac{4}{9}\right)^{\frac{3}{2}}$ (k) $25^{-\frac{3}{2}}$ (l) $169^{-\frac{1}{2}}$

(m) $\left(\dfrac{25}{144}\right)^{-\frac{1}{2}}$ (n) $(-32)^{\frac{1}{5}}$ (o) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{3}}$ (p) $\dfrac{32^{\frac{3}{5}}}{32^{\frac{2}{5}}}$

(q) $(\sqrt[3]{5})^3$ (r) $\dfrac{8^{\frac{2}{3}}}{8^{\frac{5}{3}}}$ (s) $5^{\frac{2}{5}} \cdot 5^{\frac{8}{5}}$ (t) $(\sqrt[7]{6})^{-14}$

(u) $\sqrt[3]{-1}$ (v) $\sqrt{-25}$ (w) $\sqrt[3]{-27}$ (x) $\sqrt[4]{-16}$

(y) $\sqrt[4]{16}$ (z) $\sqrt[5]{-32}$

9. Simplify and eliminate negative exponents. Assume that all letters denote positive numbers.

(a) $x^{\frac{2}{3}} \cdot x^{\frac{4}{3}}$

(b) $a^{\frac{3}{5}} \cdot a^{\frac{12}{5}}$

(c) $(9x)^{\frac{1}{2}} \cdot (4x^{\frac{1}{4}})$

(d) $((2b)^{\frac{2}{9}})^3 \cdot (2b)^{\frac{1}{3}}$

(e) $\dfrac{x^{\frac{3}{2}} x^{\frac{1}{2}}}{x^{\frac{5}{2}}}$

(f) $\dfrac{y^{\frac{4}{3}}((3y)^{\frac{2}{3}})^2}{y^{\frac{2}{3}}}$

(g) $(27z^3)^{-\frac{2}{3}}$

(h) $(x^5y^4)^{-\frac{1}{2}}$

(i) $(-8x^6y^{-18})^{-\frac{1}{3}}$

(j) $ba^{-\frac{3}{2}}(b^{\frac{3}{2}})^{\frac{4}{3}}$

(k) $\dfrac{(x^{\frac{3}{2}}y^{-1})^{-\frac{1}{2}}}{x^0 y^{\frac{3}{2}} x^{\frac{5}{4}} y^{-\frac{7}{4}}}$

(l) $y^{-1}(yx^{\frac{1}{2}})^{\frac{2}{3}}$

(m) $\left(\dfrac{a^{\frac{3}{2}}}{b^{-\frac{1}{2}}}\right)^4 \left(\dfrac{a^{-2}}{b^3}\right)$

(n) $\left(\dfrac{x^6y^{-3}}{27y^{\frac{3}{5}}}\right)^{-\frac{1}{3}}$

(o) $\left(\dfrac{2x^3y^{\frac{2}{3}}}{z^{\frac{1}{2}}}\right)\left(\dfrac{z^{-3}y^9}{8y^4}\right)^{\frac{1}{3}}$

(p) $\left(\dfrac{4x^3y^4}{x^2y^{\frac{9}{2}}}\right)^{-\frac{1}{2}}$

(q) $\left(\dfrac{s^8t^{-4}}{16t^{\frac{4}{3}}}\right)^{-\frac{1}{4}}$

Answers

1. (a) 81 (b) -81 (c) $-1/8$ (d) -32 (e) -32 (f) 1 (g) $1/3$ (h) $1/9$
(i) $25/4$ (j) 125 (k) 25 (l) $1/25$ (m) 64 (n) 1000 (o) 10,000

2. (a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) $5\sqrt[3]{2}$ (d) 3 (e) $3\sqrt[5]{2}$ (f) $3\sqrt[4]{2}$ (g) $\frac{3\sqrt{3}}{4}$ (h) $\frac{\sqrt{2}}{2}$
(i) $15\sqrt[3]{3}$ (j) $2\sqrt{3}$ (k) $8\sqrt{7}$ (l) 4 (m) $5\sqrt[3]{4}$ (n) $2\sqrt[4]{21}$ (o) 3 (p) $5\sqrt[3]{12}$
(q) $\frac{1}{3\sqrt[3]{9}}$ (r) $\frac{1}{2}$ (s) 2 (t) $3\sqrt[3]{25}$

3. (a) x^{13} (b) $4x^6$ (c) x^2 (d) y^{15} (e) $27x^3$ (f) y^4 (g) $\frac{1}{z^5}$ (h) $\frac{1}{x^4}$ (i) x^5
(j) $\frac{1}{y^6}$ (k) w (l) $\frac{1}{x^4}$ (m) b^4 (n) z^{16} (o) $\frac{x^7}{9}$ (p) y^9 (q) $-8b^{18}$ (r) $\frac{72}{x^2}$

4. (a) $\frac{x^5y^3}{7}$ (b) $\frac{7y}{4z^4}$ (c) $8y^2$ (d) $\frac{6}{x^5w^3}$ (e) $64w^{15}$ (f) y^3 (g) $1728x^9$ (h) $\frac{y^6}{4}$
(i) $\frac{729}{512z^{15}}$ (j) $\frac{1}{x^3y}$ (k) $\frac{a^{18}}{b^{12}}$ (l) $\frac{x^{11}}{yz^6}$ (m) $\frac{b^{15}}{a^8}$ (n) $\frac{9x^6y}{2}$ (o) w^2v^9 (p) $\frac{4x^9}{y^{13}}$
(q) $\frac{9}{w^2x^6}$ (r) $\frac{1}{8x^6y^9}$ (s) $\frac{c^4}{a^2b^4}$ (t) $\frac{25y^6}{x^{10}}$ (u) $\frac{27r^3s^{15}}{8t^3}$

5. (a) $x^{\frac{6}{5}}$ (b) $x^{\frac{3}{8}}$ (c) $x^{-\frac{5}{2}}$ (d) $x^{-\frac{4}{3}}$ (e) $x^{\frac{1}{12}}$ (f) $x^{-\frac{1}{10}}$

6. (a) $x^{\frac{11}{2}}$ (b) $x^{-\frac{22}{7}}$ (c) $x^{-\frac{12}{5}}$ (d) $x^{-\frac{19}{15}}$ (e) $x^{\frac{8}{27}}$ (f) $x^{-\frac{5}{2}}$ (g) x^2 (h) $x^{-\frac{3}{2}}$
(i) $x^{\frac{7}{3}}$ (j) $x^{\frac{3}{5}}$ (k) $x^{-\frac{2}{3}}$ (l) $x^{\frac{5}{4}}$

7. The true equations are (b), (c), (f), (i) and (j).

8. (a) 2 (b) -2 (c) -1 (d) 3 (e) $\frac{1}{3}$ (f) 4 (g) $\frac{3}{2}$ (h) $\frac{25}{16}$ (i) $\frac{5}{9}$ (j) $\frac{8}{27}$
(k) $\frac{1}{125}$ (l) $\frac{1}{13}$ (m) $\frac{12}{5}$ (n) -2 (o) 2 (p) 2 (q) 5 (r) $\frac{1}{8}$ (s) 25
(t) $\frac{1}{36}$ (u) -1 (v) Undefined. (w) -3 (x) Undefined. (y) 2 (z) -2

9. (a) x^2 (b) a^3 (c) $12x^{\frac{3}{4}}$ (d) $2b$ (e) $\frac{1}{x^{\frac{1}{2}}}$ (f) $3^{\frac{4}{3}}y^2$ (g) $\frac{1}{9z^2}$ (h) $\frac{1}{x^{\frac{5}{2}}y^2}$ (i) $\frac{-y^6}{2x^2}$
(j) $\frac{b^3}{a^{\frac{3}{2}}}$ (k) $\frac{y^{\frac{3}{4}}}{x^2}$ (l) $\frac{x^{\frac{1}{3}}}{y^{\frac{1}{3}}}$ (m) $\frac{a^4}{b}$ (n) $\frac{3y^{\frac{6}{5}}}{x^2}$ (o) $\frac{x^3y^{\frac{7}{3}}}{z^{\frac{3}{2}}}$ (p) $\frac{y^{\frac{1}{4}}}{2x^{\frac{1}{2}}}$ (q) $\frac{2t^{\frac{4}{3}}}{s^2}$