1. Factor completely by finding a common factor among the terms.

(a) $3x^3 - x$ (b) $2x^3 + 6x^4 - 4x^5$ (c) $z(z+2) - 4(z+2)$

(d) $(x+y)^3 - 3(x+y)^2$ (e) $3x^2y + 2x^2y^2 - 6x^3y$ (f) $5a^3b^2 + 25a^5b - 60a^2b^3$

2. Factor completely.

(a) $(2x-1)(x+3) - 4(2x-1)$ (b) $t^2(t-5) + t - 5$ (c) $w^2 - 121$

(d) $49 - 4t^2$ (e) $81t^4 - 16$ (f) $9z^2 - 64y^4$

(g) $(y+3)^2 - 4y^2$ (h) $(x+h)^3 - (x+h)$ (i) $y^2 - 24y + 144$

(j) $25t^2 + 10t + 1$ (k) $12x^3 - 36x^2 + 27x$ (l) $m^4 + 10m^2 + 25$

(m) $27 - 8x^3$ (n) $t^6 + t^3$ (o) $x^2 - 5x - 14$

(p) $y^2 - 12y + 27$ (q) $3t^2 + 16t + 5$ (r) $6x^2 - 23x + 20$

(s) $35 + 2m - m^2$ (t) $7w - 2w^2 - 3$ (u) $3m^3 + 9m^2 - 12m$

(v) $x^4 + x^2 - 20$ (w) $4(t^2-1)^2 + 3(t^2-1) - 10$ (x) $x^3 - 5x^2 - 9x + 45$

(y) $3t^2 + t - 3 - t^3$

3. Factor the following trinomials.

(a) $x^2 + 6x + 8$ (b) $y^2 + 7y + 12$ (c) $y^2 + 4y - 12$

(d) $3a^2 - 10a + 8$ (e) $2b^2 + 2b - 24$ (f) $3s^2 - 9s + 6$

(g) $(2t + 1)^2 + 6(2t + 1) + 8$ (h) $(x + y)^2 - 2(x + y) - 8$

4. Factor the following expressions completely. Recognize differences or sums of squares/cubes.

(a) $16x^2 - 9$ (b) $(y - 2)^2 - 25$ (c) $8z^3 + x^3$

(d) $x^3 - y^6$ (e) $27x^3 - 8y^3$ (f) $1 + 1000x^3$

(g) $z^2 - 9z + 20$ (h) $y^2 + 10y + 25$ (i) $16x^2 - 24x + 9$

(j) $x^4 - 9x^2$ (k) $x^2 - 10$ (l) $x^3 + 27$

(m) $1 - x^3$ (n) $x^3 - 8$ (o) $x^4 - 81$

(p) $9 - x^4$ (q) $8x^3 - 27$ (r) $64x^8 + 27x^5$

5. Factor the following expressions by grouping the terms.

(a) $x^3 + 5x^2 + x + 5$ (b) $4x^3 - x^2 + 8x - 2$ (c) $2x^3 + x^2 + 2x + 1$

(d) $24x^3 + 8x^2 + 3x + 1$ (e) $x^3 + x^2 + x + 1$ (f) $x^5 + x^4 + x + 1$

Answers

1. (a) $x(3x^2-1)$ (b) $2x^3(1+3x-2x^2)$ (c) $(z+2)(z-4)$ (d) $(x+y)^2((x+y)-3)$
   (e) $x^2y(3+2y-6x)$ (f) $5a^2b(ab+5a^3-12b^2)$

2. (a) $(2x-1)(x-1)$ (b) $(t-5)(t^2+1)$ (c) $(w-11)(w+11)$
   (d) $(7-2t)(7+2t)$ (e) $(3t-2)(3t+2)(9t^2+4)$ (f) $(3z-8y^2)(3z+8y^2)$
   (g) $-3(y-3)(y+1)$ (h) $(x+h)(x+h-1)(x+h+1)$
   (i) $(y-12)^2$ (j) $(5t+1)^2$ (k) $3x(2x-3)^2$
   (l) $(m^2+5)^2$ (m) $(3-2x)(9+6x+4x^2)$ (n) $t^3(t+1)(t^2-t+1)$
   (o) $(x-7)(x+2)$ (p) $(y-9)(y-3)$ (q) $(3t+1)(t+5)$
   (r) $(2x-5)(3x-4)$ (s) $(7-m)(5+m)$ (t) $(-2w+1)(w-3)$
   (u) $3m(m-1)(m+4)$ (v) $(x-2)(x+2)(x^2+5)$ (w) $(2t-3)(2t+3)(t^2+1)$
   (x) $(x-3)(x+3)(x-5)$ (y) $(t-3)(1-t)(1+t)$ (z) $(y^2-y+3)(y^2+y+3)$

3. (a) $(x+4)(x+2)$ (b) $(y+4)(y+3)$ (c) $(y+6)(y-2)$ (d) $(3a-4)(a-2)$
   (e) $2(b+4)(b-3)$ (f) $3(s-1)(s-2)$ (g) $(2t+5)(2t+3)$ (h) $(x+y+2)(x+y-4)$

4. (a) $(4x+3)(4x-3)$ (b) $(y+3)(y-7)$ (c) $\left(x^2-2xz+4z^2\right)(x+2z)$
   (d) $-\left(y^4+xy^2+x^2\right)\left(y^2-x\right)$ (e) $\left(9x^2+6xy+4y^2\right)(3x-2y)$ (f) $\left(100x^2-10x+1\right)(10x+1)$
   (g) $(z-4)(z-5)$ (h) $(y+5)^2$ (i) $(4x-3)^2$
   (j) $x^2(x-3)(x+3)$ (k) $(x-\sqrt{10})(x+\sqrt{10})$ (l) $(x+3)(x^2-3x+9)$
   (m) $(1-x)(x^2+x+1)$ (n) $(x-2)(x^2+2x+4)$ (o) $(x-3)(x+3)(x^2+9)$
   (p) $(\sqrt{3}-x)(x+\sqrt{3})(x^2+3)$ (q) $(2x-3)(4x^2+6x+9)$ (r) $x^5(4x+3)(16x^2-12x+9)$

5. (a) $\left(x^2+1\right)(x+5)$ (b) $\left(x^2+2\right)(4x-1)$ (c) $\left(x^2+1\right)(2x+1)$
   (d) $\left(8x^2+1\right)(3x+1)$ (e) $\left(x^2+1\right)(x+1)$ (f) $\left(x^4+1\right)(x+1)$