1. Describe how the graph of g can be obtained by the graph of f.

(a) $f(x) = x^2$, $g(x) = x^2 - 2$

(b) $f(x) = x^2$, $g(x) = x^2 + 2$

(c) $f(x) = x^2$, $g(x) = (x + 4)^2$

(d) $f(x) = x^2$, $g(x) = (x - 3)^2$

(e) $f(x) = x^2$, $g(x) = (x - 1)^2 + 4$

(f) $f(x) = x^2$, $g(x) = -x^2$

(g) $f(x) = x^2$, $g(x) = \dfrac{x^2}{3}$

(h) $f(x) = x^2$, $g(x) = \dfrac{x^2}{2} + 6$

2. Draw the graph of $y = x^2$ for reference, and graph the following functions.

(a) $g(x) = x^2 + 3$

(b) $g(x) = (2x)^2$

(c) $g(x) = -4(x^2)$

(d) $g(x) = (x - 2)^2$

(e) $g(x) = \dfrac{1}{3}x^2 + 2$

(f) $g(x) = \dfrac{1}{2}(x - 3)^2 - 2$

3. Do the following steps to each function below.

(i) Express $f(x)$ in vertex form. (Hint: complete the square.)

(ii) Find the vertex and the x and y-intercepts of f.

(iii) Sketch the graph of f.

(iv) Give the domain and range of f.

(a) $f(x) = x^2 - 3x + 2$

(b) $f(x) = x^2 - x + 5$

(c) $f(x) = x^2 + 7x - 8$

(d) $f(x) = x^2 - 8x + 3$

(e) $f(x) = -x^2 + 5x + 6$

(f) $f(x) = -x^2 - 2x + 7$

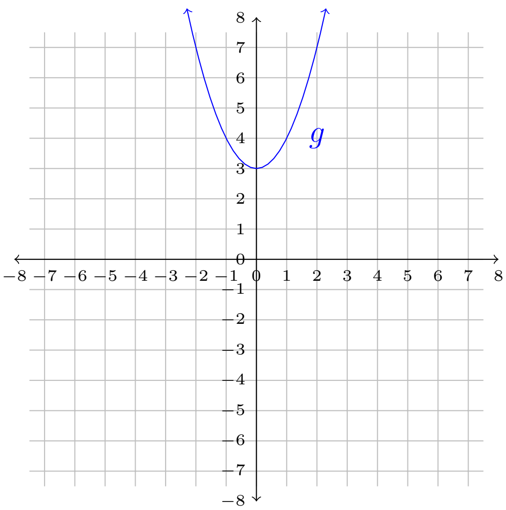
(g) $f(x) = 2x^2 - 5x + 2$

(h) $f(x) = -3x^2 + 2x - 8$

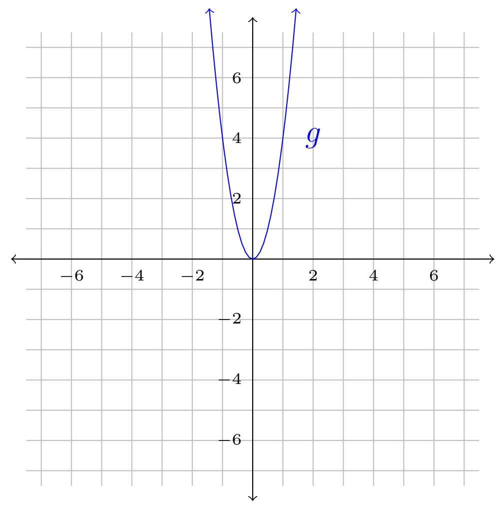
Answers

1. The graph of $g(x)$ can be obtained from the graph of $f(x)$ by...
   (a) ...shifting down 2 units.
   (b) ...shifting up 2 units.
   (c) ...shifting left 4 units.
   (d) ...shifting right 3 units.
   (e) ...shifting to the right 1 unit and up 4 units.
   (f) ...reflecting over the x-axis.
   (g) ...compressing in the y-direction by a factor of $\frac{1}{3}$.
   (h) ...compressing in the y-direction by a factor of $\frac{1}{2}$ and then shifting up 6 units.

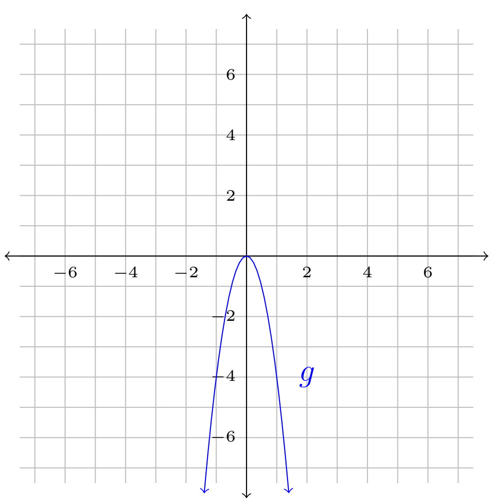
2. (a)



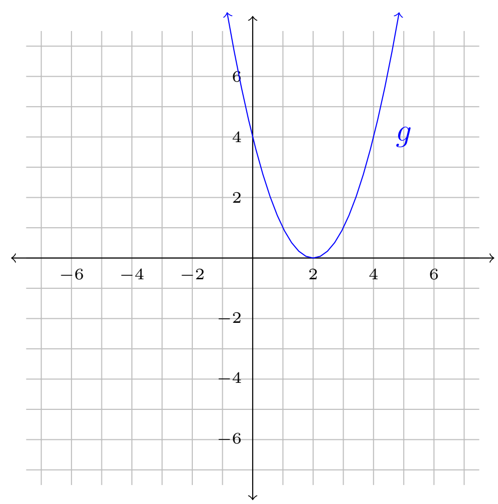
(b)



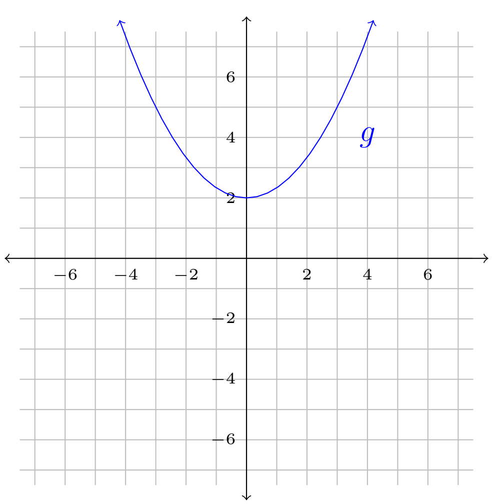
(c)



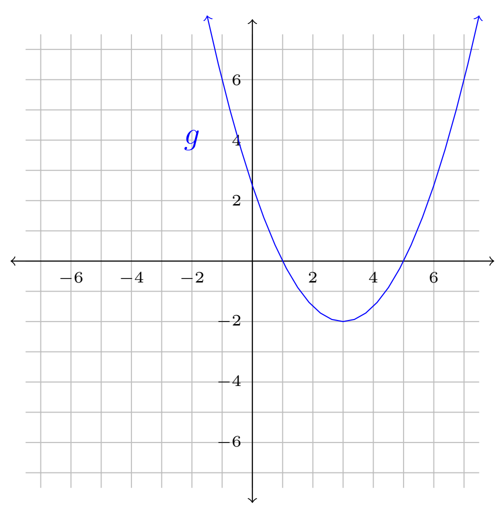
(d)



(e)



(f)

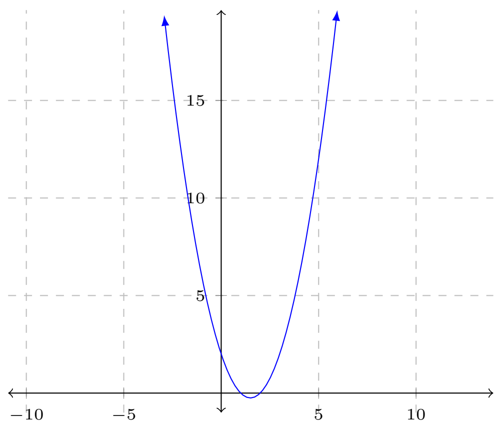


3. (a) $f(x) = x^2 - 3x + 2$

(i) $f(x) = (x - \frac{3}{2})^2 - \frac{1}{4}$

(ii) Vertex: $\left(\frac{3}{2}, -\frac{1}{4}\right)$,
x-intercepts: $\{1, 2\}$,
y-intercept: 2.

(iii)



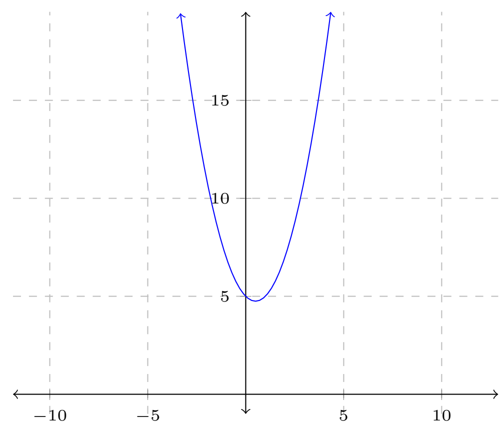
(iv) Domain: $\mathbb{R}$, range: $\left[-\frac{1}{4}, \infty\right)$.

(b) $f(x) = x^2 - x + 5$

(i) $f(x) = (x - \frac{1}{2})^2 + \frac{19}{4}$

(ii) Vertex: $\left(\frac{1}{2}, \frac{19}{4}\right)$,
no real x-intercepts,
y-intercept: 5.

(iii)



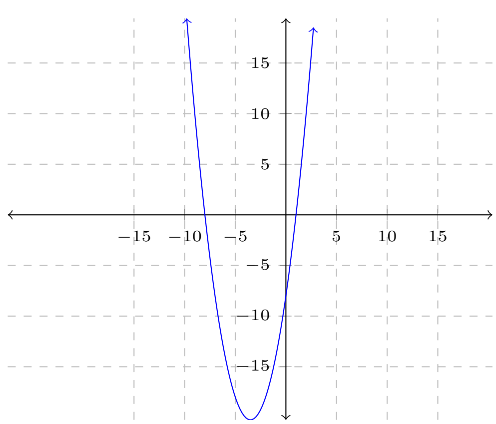
(iv) Domain: $\mathbb{R}$, range: $\left[\frac{19}{4}, \infty\right)$.

(c) $f(x) = x^2 + 7x - 8$

(i) $f(x) = (x + \frac{7}{2})^2 - \frac{81}{4}$

(ii) Vertex: $\left(-\frac{7}{2}, -\frac{81}{4}\right)$,
x-intercepts: $\{-8, 1\}$,
y-intercept: -8.

(iii)



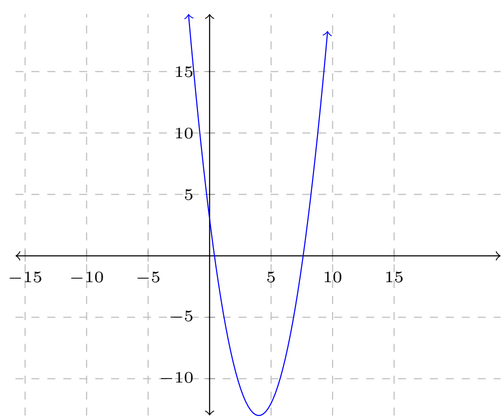
(iv) Domain: $\mathbb{R}$, range: $\left[-\frac{81}{4}, \infty\right)$.

(d) $f(x) = x^2 - 8x + 3$

(i) $f(x) = (x - 4)^2 - 13$

(ii) Vertex: $(4, -13)$,
x-intercepts: $\left\{4 - \sqrt{13}, 4 + \sqrt{13}\right\}$,
y-intercept: 3.

(iii)



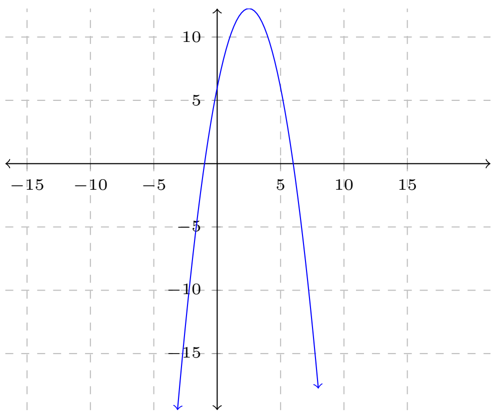
(iv) Domain: $\mathbb{R}$, range: $[-13, \infty)$.

(e) $f(x) = -x^2 + 5x + 6$

(i) $f(x) = -(x - \frac{5}{2})^2 + \frac{49}{4}$

(ii) Vertex: $\left(\frac{5}{2}, \frac{49}{4}\right)$,
x-intercepts: $\{-1, 6\}$,
y-intercept: 6.

(iii)



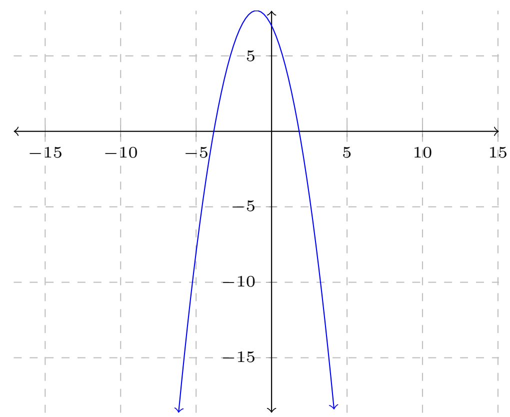
(iv) Domain: $\mathbb{R}$, range: $\left(-\infty, \frac{49}{4}\right]$.

(f) $f(x) = -x^2 - 2x + 7$

(i) $f(x) = -(x + 1)^2 + 8$

(ii) Vertex: $(-1, 8)$,
x-intercepts: $\left\{-1 - 2\sqrt{2}, -1 + 2\sqrt{2}\right\}$,
y-intercept: 7.

(iii)



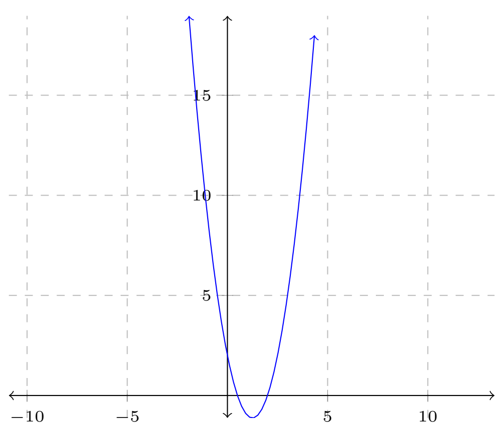
(iv) Domain: $\mathbb{R}$, range: $(-\infty, 8]$.

(g) $f(x) = 2x^2 - 5x + 2$

(i) $f(x) = 2(x - \frac{5}{4})^2 - \frac{9}{8}$

(ii) Vertex: $\left(\frac{5}{4}, -\frac{9}{8}\right)$,
x-intercepts: $\left\{\frac{1}{2}, 2\right\}$,
y-intercept: 2.

(iii)



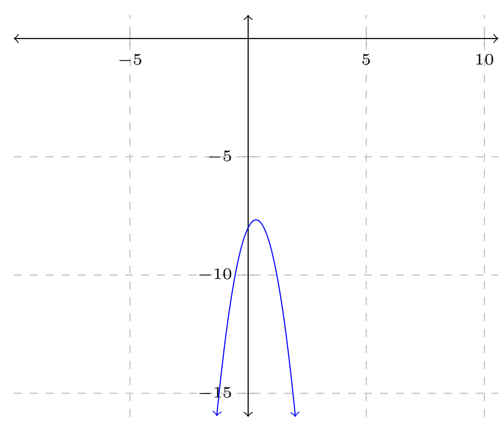
(iv) Domain: $\mathbb{R}$, range: $\left[-\frac{9}{8}, \infty\right)$.

(h) $f(x) = -3x^2 + 2x - 8$

(i) $f(x) = -3(x - \frac{1}{3})^2 - \frac{23}{3}$

(ii) Vertex: $\left(\frac{1}{3}, -\frac{23}{3}\right)$,
no real x-intercepts,
y-intercept: -8.

(iii)



(iv) Domain: $\mathbb{R}$, range: $\left(-\infty, -\frac{23}{3}\right]$.