1. Give the domain and range of the following functions in interval notation.

(a) $f(x) = e^x$ (b) $f(x) = 5^x$ (c) $f(x) = 2^x - 3$ (d) $f(x) = -4^x$

(e) $f(x) = 2^{x-3}$ (f) $f(x) = 2 + e^x$ (g) $f(x) = -4^{3x} + 5$ (h) $f(x) = 1 - e^{2x}$

(i) $f(x) = e^{x-2}$ (j) $f(x) = e^{x+1} - 3$ (k) $f(x) = \ln x$ (l) $f(x) = \log_3 x$

(m) $f(x) = \ln(x-2)$ (n) $f(x) = \log_3(x-1)$ (o) $f(x) = \ln(x+5)$ (p) $f(x) = 2 + \ln x$

2. Express the following natural logarithmic equations in exponential form.

(a) $\ln 3 = x$ (b) $\ln 18 = x - 5$ (c) $\ln 12 = x + 6$ (d) $\ln 1 = x - 8$

(e) $\ln 8 = x + 2$ (f) $\ln 5 = x + 3$ (g) $\ln 13 = x - 8$ (h) $\ln 12 = x + 6$

3. Express the following equations in natural logarithmic form.

(a) $e^{x-6} = 19$ (b) $e^{x+2} = 12$ (c) $e^{x-7} = 12$ (d) $e^{x+10} = 11$

(e) $e^{x+5} = 2$ (f) $e^{x+7} = 10$ (g) $e^{x-7} = 20$ (h) $e^{x-8} = 5$

4. Simplify the following expressions.

(a) $e^{\ln 2}$ (b) $e^{2\ln 3}$ (c) $e^{\frac{1}{2}\ln 5}$ (d) $e^{\ln\left(\frac{2}{3}\right)}$ (e) $e^{3\ln \pi}$

(f) $e^{\ln x}$ (g) $\ln 1$ (h) $\ln e^{\sqrt{7}}$ (i) $\ln(\ln(e^{e^2}))$ (j) $\ln e^x$

(k) $\ln e^{3x+2}$ (l) $\ln e$ (m) $\ln \dfrac{1}{e^3}$ (n) $\ln \sqrt{e^4}$ (o) $\ln \dfrac{1}{\sqrt[3]{e^2}}$

5. Use the laws of logarithms to expand the expressions.

(a) $\ln(6x)$ (b) $\ln(ab)^6$ (c) $\ln \sqrt{xy}$

(d) $\ln\left(\dfrac{x^5}{2y^2}\right)$ (e) $\ln\left(\dfrac{x^3}{\sqrt{2x}}\right)$ (f) $\ln \sqrt[4]{a^2+b^2}$

(g) $\ln\left(\dfrac{x}{\sqrt[3]{x-1}}\right)$ (h) $\ln(x^2-16)$ (i) $\ln(x^3+8)$

6. Combine into a single logarithm.

   (a) $\ln x + 2\ln y$

   (b) $\frac{1}{2}\ln x - 3\ln y$

   (c) $3\ln x - 2\ln(x+1)$

   (d) $3\ln x - \frac{1}{3}\ln(x^2+1) + 2\ln(x-1)$

   (e) $\ln(x^2-4) - \ln(x-2)$

   (f) $2(\ln a + 2\ln b - 3\ln c)$

   (g) $4\ln x + \ln(x^3-27) - \ln(x-3)$

   (h) $2\ln x - \ln y - 3\ln z$

7. Plot the graph of $y = e^x$ as a reference, and then plot the following functions. State the domain and range, and include the asymptotes in your plot.

   (a) $f(x) = e^{2x}$

   (b) $f(x) = e^{\frac{1}{2}x}$

   (c) $f(x) = e^{-x} - 1$

   (d) $f(x) = -e^x + 2$

   (e) $f(x) = -e^{-\frac{1}{3}x}$

   (f) $f(x) = -2e^{x+3}$

8. Plot the graph of $y = \ln x$ as a reference, and then plot the following functions. State the domain and range, and include the asymptotes in your plot.

   (a) $f(x) = \ln 2x$

   (b) $f(x) = \ln\left(\frac{1}{2}x\right)$

   (c) $f(x) = \ln(x-1)$

   (d) $f(x) = \frac{1}{2}\ln x$

Answers

1. (a) Domain: $(-\infty, \infty)$, Range: $(0, \infty)$
   (b) Domain: $(-\infty, \infty)$, Range: $(0, \infty)$
   (c) Domain: $(-\infty, \infty)$, Range: $(-3, \infty)$
   (d) Domain: $(-\infty, \infty)$, Range: $(-\infty, 0)$
   (e) Domain: $(-\infty, \infty)$, Range: $(0, \infty)$
   (f) Domain: $(-\infty, \infty)$, Range: $(2, \infty)$
   (g) Domain: $(-\infty, \infty)$, Range: $(-\infty, 5)$
   (h) Domain: $(-\infty, \infty)$, Range: $(-\infty, 1)$
   (i) Domain: $(-\infty, \infty)$, Range: $(0, \infty)$
   (j) Domain: $(-\infty, \infty)$, Range: $(-3, \infty)$
   (k) Domain: $(0, \infty)$, Range: $(-\infty, \infty)$
   (l) Domain: $(0, \infty)$, Range: $(-\infty, \infty)$
   (m) Domain: $(2, \infty)$, Range: $(-\infty, \infty)$
   (n) Domain: $(1, \infty)$, Range: $(-\infty, \infty)$
   (o) Domain: $(-5, \infty)$, Range: $(-\infty, \infty)$
   (p) Domain: $(0, \infty)$, Range: $(-\infty, \infty)$

2. (a) $e^x = 3$
   (b) $e^{x-5} = 18$
   (c) $e^{x+6} = 12$
   (d) $e^{x-8} = 1$
   (e) $e^{x+2} = 8$
   (f) $e^{x+3} = 5$
   (g) $e^{x-8} = 13$
   (h) $e^{x+6} = 12$

3. (a) $x - 6 = \ln 19$
   (b) $x + 2 = \ln 12$
   (c) $x - 7 = \ln 12$
   (d) $x + 10 = \ln 11$
   (e) $x + 5 = \ln 2$
   (f) $x + 7 = \ln 10$
   (g) $x - 7 = \ln 20$
   (h) $x - 8 = \ln 5$

4. (a) 2
   (b) 9
   (c) $\sqrt{5}$
   (d) $\frac{2}{3}$
   (e) π^3
   (f) x
   (g) 0
   (h) $\sqrt{7}$
   (i) 2
   (j) x
   (k) $3x + 2$
   (l) 1
   (m) -3
   (n) 2
   (o) $-\frac{2}{3}$

5. (a) $\ln 6 + \ln x$
   (b) $6 \ln a + 6 \ln b$
   (c) $\frac{1}{2} \ln x + \frac{1}{2} \ln y$
   (d) $5 \ln x - \ln 2 - 2 \ln y$
   (e) $3 \ln x - \frac{1}{2} \ln 2 - \frac{1}{2} \ln x$
   (f) $\frac{1}{4} \ln(a^2 + b^2)$
   (g) $\ln x - \frac{1}{3} \ln(x - 1)$
   (h) $\ln(x - 4) + \ln(x + 4)$
   (i) $\ln(x + 2) + \ln(x^2 - 2x + 4)$

6. (a) $\ln(xy^2)$
   (b) $\ln\left(\frac{\sqrt{x}}{y^3}\right)$
   (c) $\ln\left(\frac{x^3}{(x+1)^2}\right)$
   (d) $\ln\left(\frac{x^3(x-1)^2}{\sqrt[3]{x^2+1}}\right)$

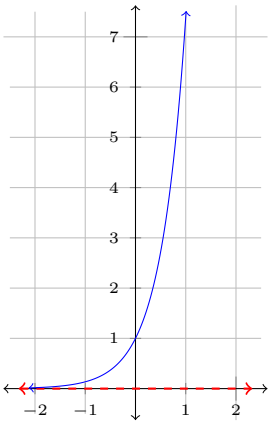
(e) $\ln(x+2)$

(f) $\ln\left(\dfrac{a^2b^4}{c^6}\right)$

(g) $\ln(x^4(x^2-3x+9))$

(h) $\ln\left(\dfrac{x^2}{yz^3}\right)$

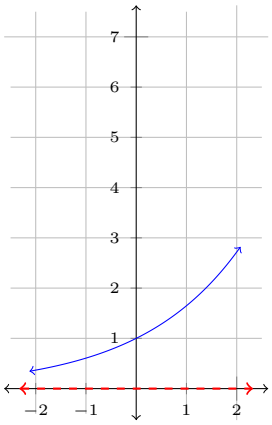
7. (a)



Domain: $\mathbb{R}$

Range: $(0,\infty)$

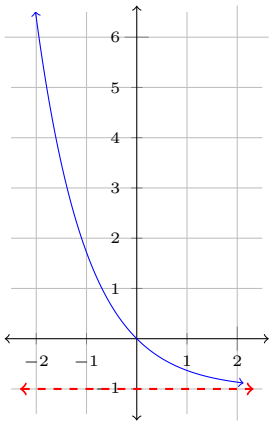
(b)



Domain: $\mathbb{R}$

Range: $(0,\infty)$

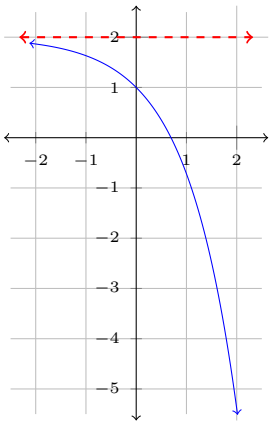
(c)



Domain: $\mathbb{R}$

Range: $(-1,\infty)$

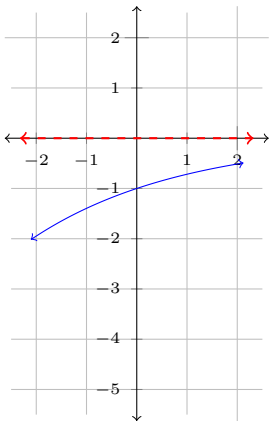
(d)



Domain: $\mathbb{R}$

Range: $(-\infty,2)$

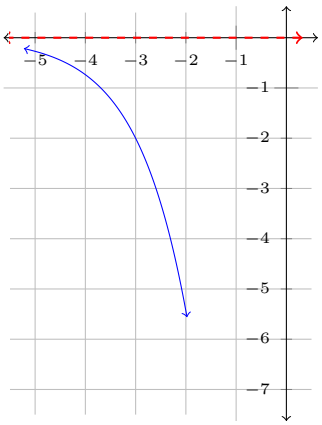
(e)



Domain: $\mathbb{R}$

Range: $(0,\infty)$

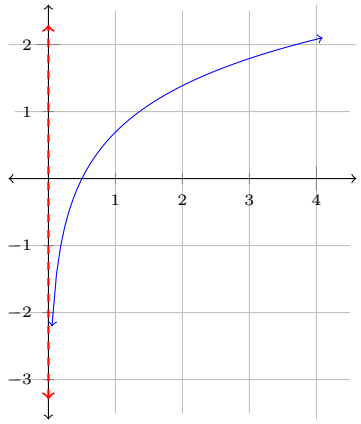
(f)



Domain: $\mathbb{R}$

Range: $(-\infty,0)$

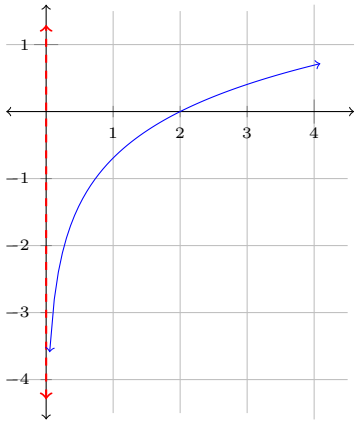
8. (a)



Domain: $(0, \infty)$

Range: $\mathbb{R}$

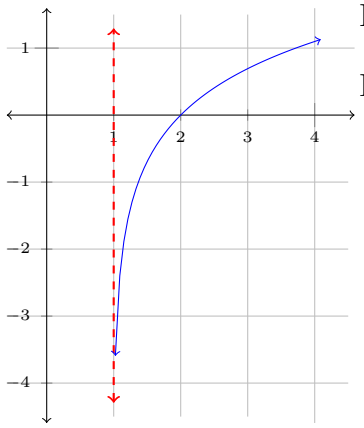
(b)



Domain: $(0, \infty)$

Range: $\mathbb{R}$

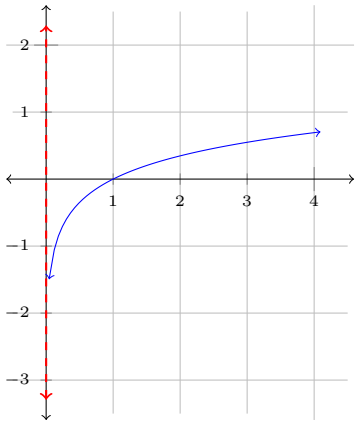
(c)



Domain: $(1, \infty)$

Range: $\mathbb{R}$

(d)



Domain: $(0, \infty)$

Range: $\mathbb{R}$