Precalculus Workshop - Equations and Inequalities

Linear Equations

To solve a linear equation, we may apply the rules below. The values a, b and c are real numbers, unless otherwise stated.

1. Addition and subtraction rules:
   (a) If $a = b$, then $a + c = b + c$.
   (b) If $a = b$, then $a - c = b - c$.
2. Multiplication and division rules:
   (a) If $a = b$, then $ca = cb$.
   (b) If $a = b$, then $\frac{a}{c} = \frac{b}{c}$ $(c \neq 0)$.
3. Switching sides:
   (a) If $a = b$, then $b = a$.

VIDEO EXAMPLE 2.1 - SOLVING LINEAR EQUATIONS

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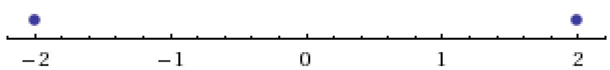
Definition of Absolute Value

Formally, the absolute value can be defined as a piece-wise function.

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

One way of interpreting the absolute value is to think of $|x|$ as the distance from the real number x to the origin 0. For example, $|-2| = 2$ and $|2| = 2$ since -2 and 2 are both two units away from the origin.



Absolute Value Equations

The general procedure to solve an equation involving a single absolute value expression is as follows:

1. Isolate the absolute value expression on one side of the equation.
2. Consider **both** the postive and negative cases of the absolute value in order to obtain all possible solutions of the equation.
3. **Check for extraneous solutions** by making a substitution into the original equation to determine if your possible solutions are valid.

VIDEO EXAMPLE 2.2 - ABSOLUTE VALUE EQUATIONS

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Quadratic Equations: Factoring

Recall that if a product of two factors is equal to zero, then at least one of the factors must be equal to zero:

1. If $ab = 0$, then either $a = 0$ or $b = 0$.

We can use this important rule to solve quadratic equations to solve a quadratic equation:

1. Rewrite your quadratic equation in standard form: $ax^2 + bx + c = 0$.
2. If possible, factor the quadratic expression on the left side of the equation. If the expression is not factorable, try a different technique.
3. Set each factor equal to zero and solve the resulting linear equation for x.

VIDEO EXAMPLE 2.3 - QUADRATIC EQUATIONS: FACTORING

The Quadratic Formula

You should have the quadratic formula memorized. If $ax^2 + bx + c = 0$, where $a \neq 0$, then all real-valued solutions are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The expression $b^2 - 4ac$ is called the discriminant. Recall:

1. $b^2 - 4ac > 0$ implies the quadratic equation has two solutions.
2. $b^2 - 4ac = 0$ implies the quadratic equation has one solution.
3. $b^2 - 4ac < 0$ implies the quadratic equation has no solutions. In this case the quadratic expression $ax^2 + bx + c$ does not factor over the real numbers and is called an irreducible quadratic.

VIDEO EXAMPLE 2.4 - QUADRATIC EQUATIONS: QUADRATIC FORMULA

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Rational Equations

Recall that if a fraction $\frac{a}{b}$ $(b \neq 0)$ is equal to zero, then its numerator must be equal to zero:

1. If $\frac{a}{b} = 0$, then $a = 0$. $(b \neq 0)$

This is an important rule when solving rational equations. To solve a rational equation:

1. Combine all rational expressions into one fraction on the left side of the equation.
2. Set the numerator equal to zero and solve the resulting equation.

VIDEO EXAMPLE 2.5 - RATIONAL EQUATIONS 1

VIDEO EXAMPLE 2.6 - RATIONAL EQUATIONS 2

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Radical Equations

Recall that the results obtained from squaring two equivalent expressions are equal:

1. If $a = b$, then $a^2 = b^2$.

This is the important rule for solving radical equations.

NOTE: If $a^2 = b^2$, it is not necessarily true that $a = b$. For example $(-1)^2 = (1)^2$, but $-1 \neq 1$. For this reason, we must check for extraneous solutions when working with radical equations.

To solve a radical equation containing one square root:

1. Isolate the square root expression on one side of the equal sign.
2. Square both sides of the equation.
3. Solve the resulting equation.
4. **Check for extraneous solutions** by making a substitution into the original equation to determine if your possible solutions are valid.

VIDEO EXAMPLE 2.7 - RADICAL EQUATIONS: ONE SQUARE ROOT

To solve a radical equation containing two square roots:

1. Isolate one square root expression on one side of the equal sign.
2. Square both sides of the equation. You should now have one radical left.
3. Isolate the second square root on one side of the equation.
4. Square both sides of the equation.
5. Solve the resulting equation.
6. **Check for extraneous solutions** by making a substitution into the original equation to determine if your possible solutions are valid.

VIDEO EXAMPLE 2.8 - RADICAL EQUATIONS: TWO SQUARE ROOTS

Precalculus Workshop - Equations and Inequalities

Interval Notation

To describe numbers that lie between endpoints, we may use the following notation.

1. For values greater than a real number a:

   (a) use $x > a$ if you do not want to include the endpoint. On the real number line, use an open circle at a.

   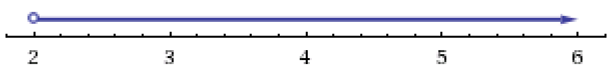

   Figure 1: The interval $x > 2$.

   (b) use $x \geq a$ if you do want to include the endpoint. Use a closed circle at a.

   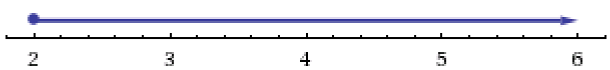

   Figure 2: The interval $x \geq 2$.

2. For values less than a real number a:

   (a) use $x < a$ if you do not want to include the endpoint. Use an open circle at a.

   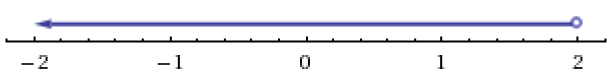

   Figure 3: The interval $x < 2$.

   (b) use $x \leq a$ if you do want to include the endpoint. Use a closed circle at a.

   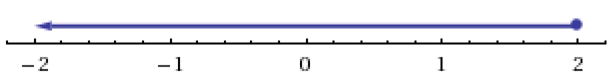

   Figure 4: The interval $x \leq 2$.

VIDEO EXAMPLE 2.9 - INTERVAL NOTATION 1

You should be comfortable using both interval notation and set notation.

1. For values greater than a real number a:

   (a) use $(a, \infty) = \{x \mid x > a\}$ if you do not want to include the endpoint. Use an open circle at a.

   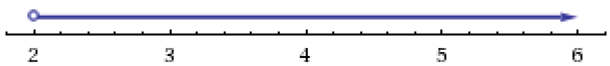

   Figure 5: The interval $(2, \infty) = \{x \mid x > 2\}$.

   (b) use $[a, \infty) = \{x \mid x \geq a\}$ if you do want to include the endpoint. Use a closed circle at a.

   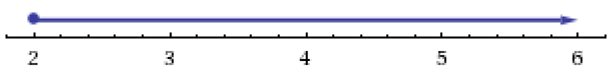

   Figure 6: The interval $[2, \infty) = \{x \mid x \geq 2\}$.

2. For values less than a real number a:

   (a) use $(-\infty, a) = \{x \mid x < a\}$ if you do not want to include the endpoint. Use an open circle at a.

   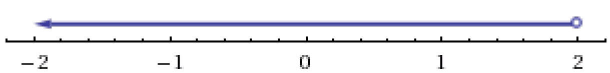

   Figure 7: The interval $(-\infty, 2) = \{x \mid x < 2\}$.

   (b) use $(-\infty, a] = \{x \mid x \leq a\}$ if you do want to include the endpoint. Use a closed circle at a.

   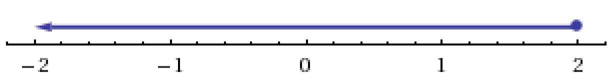

   Figure 8: The interval $(-\infty, 2] = \{x \mid x \leq 2\}$.

VIDEO EXAMPLE 2.10 - INTERVAL NOTATION 2

Precalculus Workshop - Equations and Inequalities

LINEAR INEQUALITIES

Recall the following rules for working with inequalities (where a, b, and c are real numbers):

1. Addition/Subtraction:
   (a) If $a \leq b$, then $a \pm c \leq b \pm c$.
   (b) If $a \geq b$, then $a \pm c \geq b \pm c$.
2. Multiplication/Division (if $c > 0$):
   (a) If $a \leq b$, then $ca \leq cb$ and $\frac{a}{c} \leq \frac{b}{c}$.
   (b) If $a \geq b$, then $ca \geq cb$ and $\frac{a}{c} \geq \frac{b}{c}$.
3. Multiplication/Division (if $c < 0$):
   (a) If $a \leq b$, then $ca \geq cb$ and $\frac{a}{c} \geq \frac{b}{c}$.
   (b) If $a \geq b$, then $ca \leq cb$ and $\frac{a}{c} \leq \frac{b}{c}$.
4. Reciprocals:
   (a) If $0 < a \leq b$, then $\frac{1}{a} \geq \frac{1}{b}$.
   (b) If $a \geq b > 0$, then $\frac{1}{a} \leq \frac{1}{b}$.

VIDEO EXAMPLE 2.11 - LINEAR INEQUALITIES

Precalculus Workshop - Equations and Inequalities

Absolute Value Inequalities

Recall the definition of absolute value.

If c is any positive real number, there are four absolute value inequality rules for you to know. If you interpret $|x|$ as the distance from x to 0, these inequalities should be easy to remember.

1. If $|x| < c$, then $-c < x < c$.

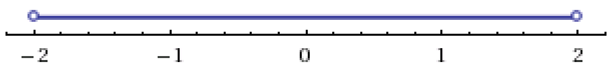

Figure 1: $|x| < 2$ means $-2 < x < 2$.

2. If $|x| > c$, then $x < -c$ or $x > c$.

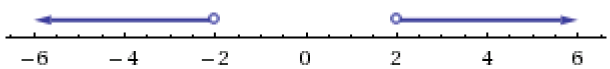

Figure 2: $|x| > 2$ means $x < -2$ or $x > 2$.

3. If $|x| \leq c$, then $-c \leq x \leq c$

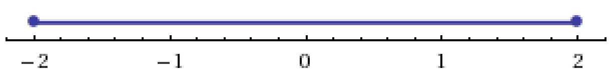

Figure 3: $|x| \leq 2$ means $-2 \leq x \leq 2$.

4. If $|x| \geq c$, then $x \leq -c$ or $x \geq c$

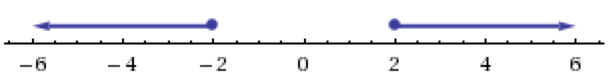

Figure 4: $|x| \geq 2$ means $x \leq -2$ or $x \geq 2$.

VIDEO EXAMPLE 2.12 - ABSOLUTE VALUE INEQUALITIES